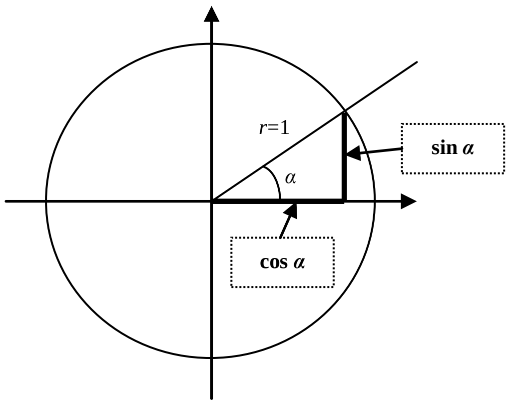
K.I.M. GONIOMETRIA E TRIGONOMETRIA *a cura di Padovan Claudio (v. 1.1)*

La circonferenza goniometrica (*r*=1)



Gradi e Radianti

$$\alpha°:\alpha^r = 180:\pi$$

con α°=angolo in Gradi-α^r= angolo in Radianti

Formule notevoli di goniometria

$$\sin^2\alpha + \cos^2\alpha = 1$$
$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$$
$$\cot\alpha = \frac{1}{\tan\alpha} = \frac{\cos\alpha}{\sin\alpha}$$
$$\sec\alpha = \frac{1}{\cos\alpha}\ ;\ \ \mathrm{cosec}\,\alpha = \frac{1}{\sin\alpha}$$
$$\sin\alpha = \pm\frac{\tan\alpha}{\sqrt{1+\tan^2\alpha}}$$
$$\cos\alpha = \pm\frac{1}{\sqrt{1+\tan^2\alpha}}$$

Archi associati

Archi complementari

$$\sin\left(\frac{\pi}{2}-\alpha\right) = \cos\alpha$$
$$\cos\left(\frac{\pi}{2}-\alpha\right) = \sin\alpha$$
$$\tan\left(\frac{\pi}{2}-\alpha\right) = \cot\alpha$$
$$\cot\left(\frac{\pi}{2}-\alpha\right) = \tan\alpha$$

$$\sin\left(\frac{\pi}{2}+\alpha\right) = \cos\alpha$$
$$\cos\left(\frac{\pi}{2}+\alpha\right) = -\sin\alpha$$
$$\tan\left(\frac{\pi}{2}+\alpha\right) = -\cot\alpha$$
$$\cot\left(\frac{\pi}{2}+\alpha\right) = -\tan\alpha$$

Archi supplementari

$$\sin(\pi-\alpha) = \sin\alpha$$
$$\cos(\pi-\alpha) = -\cos\alpha$$
$$\tan(\pi-\alpha) = -\tan\alpha$$
$$\cot(\pi-\alpha) = -\cot\alpha$$

$$\sin(\pi+\alpha) = -\sin\alpha$$
$$\cos(\pi+\alpha) = -\cos\alpha$$
$$\tan(\pi+\alpha) = \tan\alpha$$
$$\cot(\pi+\alpha) = \cot\alpha$$

Archi opposti

$$\sin(-\alpha) = -\sin\alpha$$
$$\cos(-\alpha) = \cos\alpha$$
$$\tan(-\alpha) = -\tan\alpha$$
$$\cot(-\alpha) = -\cot\alpha$$

Formule di addizione

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$$

Formule di sottrazione

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$$

Formule di duplicazione

$$\sin(2\alpha) = 2\sin\alpha\cos\alpha$$
$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha =$$
$$= 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha$$
$$\tan(2\alpha) = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

Formule di bisezione

$$\sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$
$$\cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$
$$\tan\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} =$$
$$= \frac{\sin\alpha}{1+\cos\alpha} = \frac{1-\cos\alpha}{\sin\alpha}$$

Formule parametriche

$$\sin\alpha = \frac{2\ \tan\left(\frac{\alpha}{2}\right)}{1+\tan^2\left(\frac{\alpha}{2}\right)}$$
$$\cos\alpha = \frac{1-\tan^2\left(\frac{\alpha}{2}\right)}{1+\tan^2\left(\frac{\alpha}{2}\right)}$$
$$\tan\alpha = \frac{2\ \tan\left(\frac{\alpha}{2}\right)}{1-\tan^2\left(\frac{\alpha}{2}\right)}$$

Formule di prostaferesi

$$\sin p + \sin q = 2\ \sin\frac{p+q}{2}\cos\frac{p-q}{2}$$
$$\sin p - \sin q = 2\ \sin\frac{p-q}{2}\cos\frac{p+q}{2}$$
$$\cos p + \cos q = 2\ \cos\frac{p+q}{2}\cos\frac{p-q}{2}$$
$$\cos p - \cos q = -2\ \sin\frac{p+q}{2}\sin\frac{p-q}{2}$$

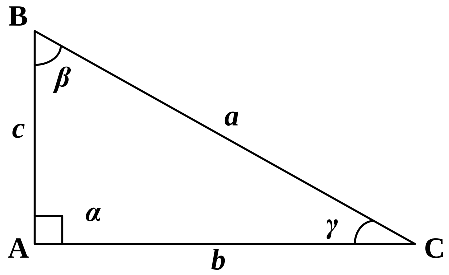
Formule di Werner

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$
$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$$
$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$

TRIGONOMETRIA

Triangoli rettangoli

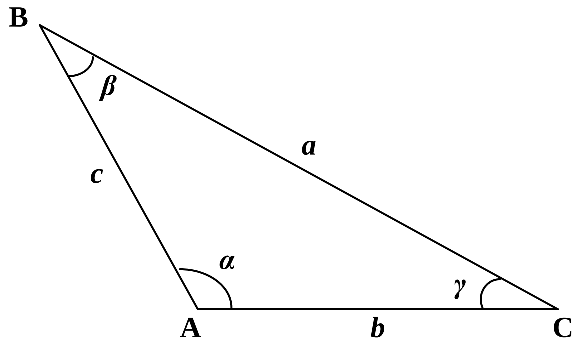
con $\alpha = 90° = \frac{\pi}{2}$



$b = a\sin\beta = a\cos\gamma$
$c = a\sin\gamma = a\cos\beta$

$b = c\tan\beta = c\cot\gamma$
$c = b\tan\gamma = b\cot\beta$

Triangoli qualsiasi



Teorema del seno

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} = 2R$$

Teorema del coseno

$$a^2 = b^2 + c^2 - 2\,b\,c\,\cos\alpha$$
$$b^2 = a^2 + c^2 - 2\,a\,c\,\cos\beta$$
$$c^2 = a^2 + b^2 - 2\,a\,b\,\cos\gamma$$

Teorema delle proiezioni

$$a = b\cos\gamma + c\cos\beta$$
$$b = c\cos\alpha + a\cos\gamma$$
$$c = a\cos\beta + b\cos\alpha$$

Area triangolo

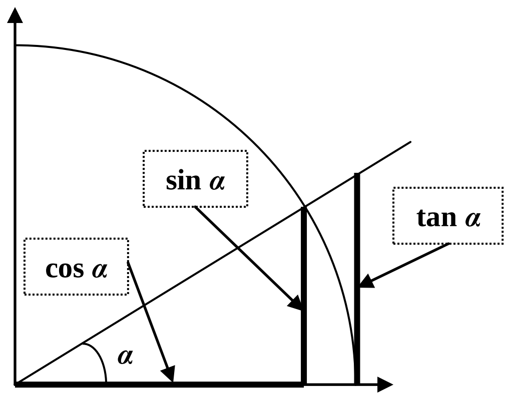
$$A = \frac{1}{2} a \cdot b \cdot \sin\gamma$$
$$A = \frac{1}{2} a \cdot c \cdot \sin\beta$$
$$A = \frac{1}{2} b \cdot c \cdot \sin\alpha$$

Teorema di Nepero: $\frac{a-b}{a+b} = \frac{\tan\frac{\alpha-\beta}{2}}{\tan\frac{\alpha+\beta}{2}}$; $\frac{b-c}{b+c} = \frac{\tan\frac{\beta-\gamma}{2}}{\tan\frac{\beta+\gamma}{2}}$; $\frac{c-a}{c+a} = \frac{\tan\frac{\gamma-\alpha}{2}}{\tan\frac{\gamma+\alpha}{2}}$

Formule di Briggs:

$$\begin{cases} \sin\left(\frac{\alpha}{2}\right) = \sqrt{\frac{(p-b)(p-c)}{bc}} \\ \cos\left(\frac{\alpha}{2}\right) = \sqrt{\frac{p(p-a)}{bc}} \\ \tan\left(\frac{\alpha}{2}\right) = \sqrt{\frac{(p-b)(p-c)}{p(p-a)}} \end{cases}$$

APPENDICE: Archi notevoli



Angolo/Arco (α)	Seno	Coseno	Tangente
$0°/0$	0	1	0
$15°/\frac{\pi}{12}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$2-\sqrt{3}$
$18°/\frac{\pi}{10}$	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\sqrt{1-\frac{2\sqrt{5}}{5}}$
$22°30'/\frac{\pi}{8}$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\sqrt{2}-1$
$30°/\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$45°/\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$60°/\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$67°30'/\frac{3\pi}{8}$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$\sqrt{2}+1$
$72°/\frac{2\pi}{5}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{5}-1}{4}$	$\sqrt{5+2\sqrt{5}}$
$75°/\frac{5\pi}{12}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$2+\sqrt{3}$
$90°/\frac{\pi}{2}$	1	0	$\pm\infty$